488. Proposed by ROGER A. JOHNSON, Western Reserve University.

If triangles are constructed on a given base, having the radii of the incircle and circumcircle in a constant ratio, determine the locus of the vertex. (Necessarily the constant ratio is not greater than $\frac{1}{2}$.)

CALCULUS.

405. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Determine the greatest quadrilateral which can be formed with the four given sides a, b, c, and d taken in order.

406. Proposed by C. N. SCHMALL, New York City.

Given $f(x+h)+f(x-h)=f(x)\cdot f(h)$, determine by Taylor's theorem or otherwise the nature of the function f.

MECHANICS.

324. Proposed by H. S. UHLER, Yale University.

A rigid body of any shape is at rest in a neutral liquid which is also at rest and has an indefinitely great volume. The body is so situated that the free surface of the liquid is tangent to it at its highest point (or points). All the space above the liquid is filled with a neutral, stagnant fluid whose density is not greater than the density of the liquid. Show that the work done in raising (pure translation) the body very slowly until the interface of the two fluids is tangent to it at its lowest point (or points) is expressible by the formula $mgh - gV(\rho_1 h_1 + \rho_2 h_2)$, where $m \equiv$ mass of body, $V \equiv$ volume of body, $\rho_1 \equiv$ mean density of lower medium in the region involved, $\rho_2 \equiv$ density of upper medium, $h_1 \equiv$ distance of center of mass of displaced liquid below the free surface in the initial position of the body, $h_2 \equiv$ elevation of center of mass of displaced fluid above interface in final position of body, and $h = h_1 + h_2$. (Neglect surface-tension, adhesion, cavities in upper portion of body, etc. This problem arose in connection with a question concerning the raising of a dense object from the bottom of a harbor to the deck of a vessel.)

325. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

The lever of a testing-machine is c feet long, and is poised on a knife-edge a feet from one end and b feet from the other, and in a horizontal line, above which the beam is symmetrical. The beam is m inches deep at the knife-edge, and tapers uniformly to a depth of n inches at each end; the width of the beam is the same throughout its length. Find the distance of the center of gravity of the beam from the knife-edge.

NUMBER THEORY.

242. Proposed by NORMAN ANNING, Chilliwack, B. C.

Find a function of n which is equal to A_k when $n \equiv k \pmod{p}$, $k = 1, 2, 3, 4, \cdots, p$.

243. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Determine the rational value of x that will render $x^3 + px^2 + qx + r$ a perfect cube. Apply the result to $x^3 - 8x^2 + 12x - 6$.

Below are given problems in Number Theory proposed between January, 1913, and January, 1915, for which no solutions have been received. Ten problems in this subject were proposed during 1915. For some of these, solutions have been received and others are doubtless under consideration by those interested. They are Nos. 227–236. While not neglecting these more recent ones, may we also have coöperation in clearing up the older list?

191. (**Incorrectly numbered 187 in the June, 1913, issue.**) **Proposed by L. E. DICKSON, University of Chicago.**

Find an amicable number triple by solving one of the equations (other than the last) in the MONTHLY, March, 1913, page 92. Note that a solution a is to be excluded if not prime to the numbers in the same line.

192. (**Incorrectly numbered 188 in the June, 1913, issue.**) **Proposed by ARTEMAS MARTIN, Washington, D. C.**

Find rational values for v, w, and x that will simultaneously satisfy the conditions:

$$(m^2 + n^2)(v^2 + w^2 + x^2)^2 - 4m^2n^2v^2 + m^2n^2(m^2 + n^2) = \square, \tag{1}$$

$$(m^2 + n^2)(v^2 + w^2 + x^2)^2 - 4m^2n^2w^2 + m^2n^2(m^2 + n^2) = \square, \tag{2}$$

$$(m^2 + n^2)(v^2 + w^2 + x^2)^2 - 4m^2n^2x^2 + m^2n^2(m^2 + n^2) = \square, \tag{3}$$

and n being known quantities.

196. (**Incorrectly numbered 192 in the September, 1913, issue.**) **Proposed by CHARLES MACAULEY, Chicago, Ill.**

Combinations containing an even number of letters are formed with the letters a, b, c, d, etc. It is required to place the letters in two columns, so that half the letters in every combination are placed in one column and the other letters of the combination in the other column, and so that all the a's stand in the same column; all the b's in the same column; all the c's in the same column, etc.

198. (**November, 1913, issue.**) **Proposed by ARTEMAS MARTIN, Washington, D. C.**

Prove that every even number is the sum of two prime numbers.

Note.—This problem has long been known and no proof has ever been given. EDITORS.

202. (**December, 1913.**) **Proposed by A. R. SCHWEITZER, Chicago, Ill.**

There exists an infinitude of systems of dyads $\{\alpha\beta\}$ in 7, 9, 11, etc., elements such that each system has the following properties: (1) if $\alpha\beta$ is in the set, then $\beta\alpha$ is not in the set; (2) for each dyad $\alpha\beta$ in the set there exists an element ξ such that $\xi\beta$ and $\alpha\xi$ are also in the set. For example, such a system is,

12,	23,	34,	45,	56,	67,	78,	89,	91
13,	24,	35,	46,	57,	68,	79,	81,	92
14,	25,	36,	47,	58,	69,	71,	82,	93
51,	62,	73,	84,	95,	16,	27,	38,	49

Investigate the existence of

I. A finite set of triads $\{\alpha\beta\gamma\}$ such that (1) if $\alpha\beta\gamma$ is in the set, then $\beta\gamma\alpha$, $\gamma\alpha\beta$ are also in the set but $\beta\alpha\gamma$ is not in the set, (2) for each triad $\alpha\beta\gamma$ in the set there exists an element ξ such that $\xi\beta\gamma$, $\alpha\xi\gamma$, $\alpha\beta\xi$ are also in the set.

II. A finite set of tetrads $\{\alpha\beta\gamma\delta\}$ such that (1) if $\alpha\beta\gamma\delta$ is in the set, then $\beta\gamma\alpha\delta$, $\gamma\alpha\beta\delta$, $\gamma\delta\alpha\beta$ are also in the set but $\beta\alpha\gamma\delta$ is not in the set, (2) for each tetrad $\alpha\beta\gamma\delta$ in the set there exists an element ξ such that $\xi\beta\gamma\delta$, $\alpha\xi\gamma\delta$, $\alpha\beta\xi\delta$, $\alpha\beta\gamma\xi$ are also in the set.

The problem for alternating n-*ads* for $n > 4$ is obvious.

205. (**February, 1914.**) **Proposed by E. T. BELL, University of Washington.**

Show that in the usual arithmetical sense the form that follows admits of composition; give the requisite transformations; and indicate how several, if not all, solutions may be found:

$$x_0^2 + nrx_1^2 + mrx_2^2 + mnx_3^2 + mnrx_4^2 + mn^2r^2x_5^2 + nr^2m^2x_6^2 + rm^2n^2x_7^2.$$

208. (**March, 1914.**) **Proposed by E. T. BELL, University of Washington.**

If an odd number is perfect it cannot be the sum of two squares.

209. (**March, 1914.**) **Proposed by R. D. CARMICHAEL, University of Illinois.**

Prove that the difference of the sixth powers of an integer cannot be the square of an integer.

211. **(April, 1914.)** **Proposed by E. T. BELL, University of Washington.**

If an odd perfect number exists, the total number of its divisors is a multiple of 2, but not of 4; or, what is the same thing, an odd perfect number must be of the form $p^{2a-1}n^2$, where p is prime and a is odd.

214. **(April, 1914.)** **Proposed by A. J. KEMPNER, University of Illinois.**

Let a be a positive integer $\geqq 2$, and let $T(n)$ denote the number of distinct divisors of the positive integer n, including both 1 and n, so that $T(1) = 1$, $T(2) = 2$, $T(3) = 2$, $T(4) = 3$, $\cdots$. Show that

$$\Sigma_{n=1}^{n=\infty} T(n)/a^n = \Sigma_{n=1}^{n=\infty} 1/(a^n - 1).$$

The special case $a = 10$ gives, as is easily seen:

$$9\sum_{n=1}^{n=\infty} \frac{T(n)}{10^n} = \frac{1}{1} + \frac{1}{11} + \frac{1}{111} + \frac{1}{1111} + \cdots.$$

217. **(May, 1914.)** **Proposed by E. T. BELL, University of Washington.**

(i) If r is a prime greater than 2, and $p \equiv 2^a r + 1$ is prime, the only solution, when n is greater than 2, of $x^n - y^n = p$, is $n = 3$, $x = 2$, $y = 1$.

(ii) The only primes that are simultaneously of the forms $4k + 1$ and $3^m - 2^m$ are 1 and 5.

(iii) Generalize (ii).

219. **(June, 1914.)** **Proposed by R. D. CARMICHAEL, University of Illinois.**

Determine whether it is possible for a polygon to have the number of its diagonals equal to a perfect fourth power.

221. **(September, 1914.)** **Proposed by T. E. MASON, Purdue University.**

Find a number x such that the sum of the divisors of x is a perfect square. [Carmichael, *Theory of Numbers*, page 17.]

222. **(October, 1914.)** **Proposed by A. H. HOLMES, Brunswick, Me.**

Find rational values for m and n such that $(m^2 + 1)/m^2 + (n^2 + 1)/n^2$ may be the square of an integer.

223. **(October, 1914.)** **Proposed by T. E. MASON, Purdue University.**

Show that

$$\frac{(rst)!}{t!(s!)^t(r!)^{st}}$$

is an integer, r, s, and t being positive integers. Generalize to the case of n integers, $r, s, t, u, \cdots$. [Carmichael, *Theory of Numbers*, page 28.]

SOLUTIONS OF PROBLEMS.

ALGEBRA.

443. **Proposed by A. M. KENYON, Purdue University.**

If p_r denote the sum of all the r-factor products that can be formed from the first n natural numbers ($p_r = 0$ for $r > n$), and if

$$D_s = \begin{vmatrix} p_1 & 1 & 0 & \cdots & 0 \\ 2p_2 & p_1 & 1 & \cdots & 0 \\ 3p_3 & p_2 & p_1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ sp_s & p_{s-1} & p_{s-2} & \cdots & p_1 \end{vmatrix}$$